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# INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

PCAC ANOMALIES AND GRAVITATION

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### ABSTRACT

The Feynman rules of electron-graviton interaction are used to calculate the graviton self-energy and the  $\pi^0 \rightarrow 2g$  matrix element, the direct analogues of the photon self-energy and  $\pi^0 \rightarrow 2\gamma$ . The lowest-order perturbation result leads to the modified PCAC relation,

$$\partial_{\alpha} j_{\alpha 5} = 2m j_5 + e^2 \epsilon_{\alpha\beta\gamma\delta} F_{\alpha\beta} F_{\gamma\delta} / 16\pi^2 + \epsilon_{\kappa\lambda\mu\nu} R_{\kappa\lambda\rho\sigma} R_{\mu\nu\rho\sigma} / 768\pi^2$$

where the anomalous terms on the right are the lowest-order non-generally covariant contributions of infinite-mass regulator fermion loops. The important question of whether the gravitational anomaly should be generally covariant and (if higher-order calculations show that this is not the case) whether compensating leptons are an absolute must, is, unfortunately, left open to the order we have worked.

## I. INTRODUCTION

Anomalous Ward-Takahashi identities have had an important impact <sup>1)</sup> on at least two branches of elementary particle physics. In the first place, they have proved relevant to the determination of pseudoscalar meson lifetimes and, in the second place, they have placed strong restrictions on the acceptability of renormalizable models <sup>2)</sup> based on spontaneous symmetry breaking <sup>3)</sup> which unify weak and electromagnetic interactions <sup>4)</sup>. This paper generalizes the triangle loop anomaly to gravitation. We find that in lowest-order perturbation theory the PCAC relation is modified to

$$\partial_\alpha j_{\alpha 5} = 2mj_5 + e^2 \epsilon_{\alpha\beta\gamma\delta} F_{\alpha\beta} F_{\gamma\delta} / 16\pi^2 + \epsilon_{\kappa\lambda\mu\nu} R_{\kappa\lambda\rho\sigma} R_{\mu\nu\rho\sigma} / 768\pi^2 \quad (A)$$

where the corrections on the right are associated with infinite-mass regulator loops.

Since the Riemann tensor  $R_{\alpha\beta\gamma\delta}$  plays the analogous role in gravity theory to the electromagnetic tensor  $F_{\alpha\beta}$ , it is interesting that the form of the gravitational anomaly for  $\pi^0 \rightarrow 2g$  parallels the form for the electromagnetic anomaly  $\pi^0 \rightarrow 2\gamma$ . An important problem, however, remains. Are there also anomalous terms in the  $\pi^0 \rightarrow 3g$ ,  $\pi^0 \rightarrow 4g$ , ... processes of a form such that they all sum together to make the expression  $\epsilon RR$  manifestly generally covariant? If this does not happen, would there be a disaster for gravity theory - e.g. emergence of a non-zero graviton self-mass or eventually a breakdown of the equivalence principle (i.e. a clash between the predictions of the theory and the Eötvös experiment)? If this happened, the existence of compensating leptons to remove the anomaly would become a necessity, not just for reasons of renormalizability, but for deeper physical reasons connected with the equivalence principle.

Unfortunately the work presented in this paper does not include  $\pi^0 \rightarrow 3g$ ,  $\pi^0 \rightarrow 4g$ , or higher processes, and we cannot make any definitive statement about the question raised above, though a few discussion remarks are presented in the last section. We do, however, consider another problem which concerns the general attitude about the interpretation of the anomalies.

